

PHYSICALLY BASED FAILURE CRITERIA FOR TRANSVERSE MATRIX CRACKING

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Abstract. A criterion for matrix failure of laminated composite plies in transverse tension and in-plane shear is developed by examining the mechanics of transverse matrix crack growth. Matrix cracks are assumed to initiate from manufacturing defects and can propagate within planes parallel to the fiber direction and normal to the ply mid-plane. Fracture mechanics models of cracks in unidirectional laminates, embedded plies and outer plies are developed to determine the onset and direction of propagation for unstable crack growth. The models for each ply configuration relate ply thickness and ply toughness to the corresponding in-situ ply strength. Calculated results for several materials are shown to correlate well with experimental results.

1. INTRODUCTION

Strength-based failure criteria are commonly used with the finite element method to predict failure events in composite structures. A large number of continuum-based criteria have been derived to relate internal stresses and experimental measures of material strength to the onset of failure. The most typical among the proposed criteria is Hashin's criterion [1] which assumes a quadratic interaction between the stresses acting on the fracture plane.

The recent World Wide Failure Exercise (WWFE) conceived and conducted by Hinton and Soden [2] provides a good assessment of the status of currently available theoretical methods for predicting material failure in fiber reinforced polymer composites. Comparison of the predictions by the WWFE participants with experimental results indicates that even when analyzing simple laminates that have been studied extensively over the past 40 years, the predictions of most theories differ significantly from the experimental observations.

The uncertainty in the prediction of initiation and progression of damage in composites has led to the undertaking of an effort at the NASA Langley Research Center to revisit existing failure theories, assess their capabilities, and to develop new theories where necessary. As a result of that effort, a set of six nonempirical criteria for predicting failure of unidirectional FRP laminates were proposed [3]. The objective

of this paper is to present a more detailed examination of the failure criterion for matrix cracking presented in Ref. [3].

2. MATRIX CRACKING AND IN-SITU STRENGTHS

Transverse matrix cracking is often considered a benign mode of failure because it normally causes such a small reduction in the overall stiffness of a structure that it is difficult to detect during a test. However, transverse matrix cracks also provide the primary leakage path for gases in pressurized vessels. Leakage is a phenomenon that is receiving considerable interest after the cancellation of the NASA X-33 launch vehicle program [4], where it was found that liquid hydrogen had leaked at cryogenic temperatures and, once the temperature increased, caused major delaminations by cryopumping.

To predict matrix cracking in a laminate subjected to in-plane shear and transverse tensile stresses, a failure criterion must be capable of calculating the "in-situ" strengths. The in-situ effect, originally detected in Parvizi's [5] tensile tests of cross-ply glass fiber reinforced plastics, is characterized by higher transverse tensile and shear strengths of a ply when it is constrained by plies with different fiber orientations in a laminate, compared with the strength of the same ply in a unidirectional laminate. The in-situ strength also depends on the number of plies clustered together, and on the fiber orientation of the constraining plies.

The orientation of the constraining plies and the number of plies clustered together also affect the crack density and the stiffness reduction of the cracked ply. Wang's [6] tests of $[0/90_n/0]$ ($n=1,2,3,4$) carbon/epoxy laminates have shown higher crack densities for thinner 90° layers. The reduction of the elastic properties of a cracked ply is normally predicted using elastic analyses of cracked plies [7, 8], or Continuum Damage Models [9-12].

The in-situ effect is illustrated in Fig. 1, where the relation between the in-situ transverse tensile strength and the total thickness of the 90° plies clustered together is represented.

Accurate in-situ strengths are necessary for any stress-based failure criterion for matrix cracking in constrained plies. Both experimental [6, 13, 14] and analytical methods [7, 15, 16] have been proposed to determine the in-situ strengths. In the following sections, the in-situ strengths are calculated using fracture mechanics solutions for the propagation of cracks in constrained plies.

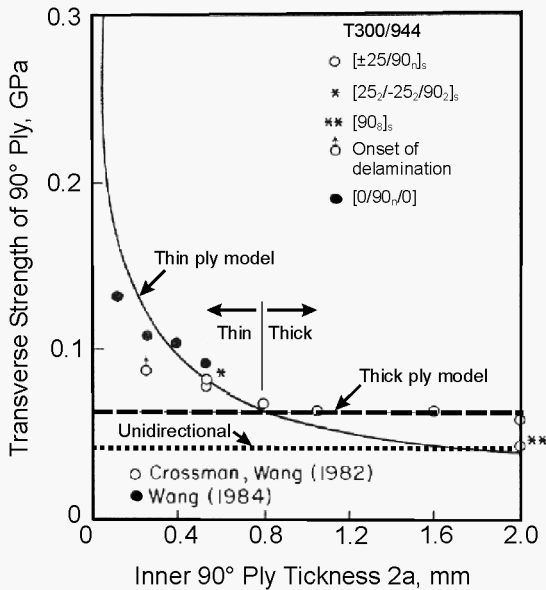


Fig. 1 – Transverse tensile strength as a function of number of plies clustered together, with models from Dvorak [15] based on experimental data from Wang [6].

Fracture Mechanics Analysis of a Cracked Ply

The failure criterion for predicting matrix cracking in a ply subjected to in-plane shear and transverse tension proposed here is based on the fracture mechanics analysis of a slit crack in a

ply, as proposed by Dvorak and Laws.[15] A slit crack represents the macroscopic effect of distributed matrix-fiber debonds that occur at the micromechanical level [6], as shown in Fig. 2. Slit cracks are the result of manufacturing defects or residual thermal stresses caused by different coefficients of thermal contraction between the matrix and the fibers. Before the slit crack grows unstably, it is idealized as lying on the 1-3 plane, as represented in Fig. 3. The crack has a length $2a_0$ across the thickness of a ply, t .

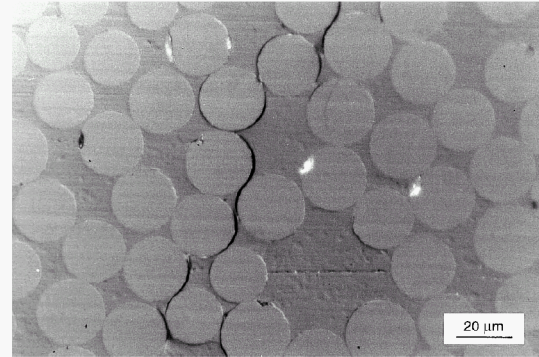


Fig. 2 – Fiber-matrix debonds coalesce to form a longer transverse crack. Ref. [17]

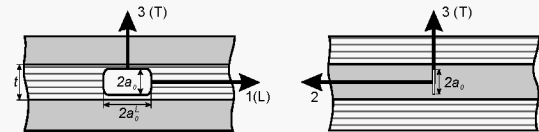


Fig. 3 – Slit crack geometry (after Dvorak[15]).

Plane stress conditions are assumed. The transverse tensile stress σ_{22} is associated with mode I loading, whereas the in-plane shear stress τ_{12} is associated with mode II loading. The crack represented in Fig. 3 can grow in the 1 (longitudinal, L) direction, in the 3 (transverse, T) direction, or in both directions.

The components of the energy release rate for the crack geometry represented in Fig. 3 were determined by Dvorak and Laws [15]. For mixed-mode loading, the energy release rate for crack growth in the T and L directions, $G(T)$ and $G(L)$, respectively, are given by:

$$\begin{aligned} G(T) &= \frac{\pi a_0}{2} (\eta_I^2 \Lambda_{22}^0 \sigma_{22}^2 + \eta_{II}^2 \Lambda_{44}^0 \tau_{12}^2) \\ G(L) &= \frac{\pi a_0}{4} (\xi_I^2 \Lambda_{22}^0 \sigma_{22}^2 + \xi_{II}^2 \Lambda_{44}^0 \tau_{12}^2) \end{aligned} \quad (1)$$

where it can be observed that the energy release rate $G(L)$ for longitudinal propagation is a function of the transverse slit size a_0 and that it is not a function of the slit length in the longitudinal direction a_0^L .

The parameters η_i , $i=I,II$ in Eq. 1 are the stress intensity reduction coefficients for propagation in the transverse direction, and the parameters ξ_i , $i=I,II$ are the reduction coefficients for propagation in the longitudinal direction. These coefficients account for the constraining effects of the adjoining layers on crack propagation: the coefficients are nearly equal to unity when $2a_0 \ll t$, and are less than unity when $a_0 \approx t$. Experimental results [14] have shown an increase in the in-situ transverse tensile strength of $[\pm\theta/90_n]_s$, $\theta=0^\circ, 30^\circ, 60^\circ$, laminates for increasing stiffness of adjoining sublaminates $\pm\theta$. This implies that the value of the parameter η_i decreases with increasing stiffness of adjoining sublaminates. Considering that a transverse crack can promote delamination between the plies, Dvorak and Laws [15] suggested that the effective value of η_i can be larger than obtained from the analysis of cracks terminating at the interface, and suggested the use of $\eta_i = \xi_i = 1$.

The parameters Λ_{ij}^0 are calculated as [15]

$$\begin{aligned} \Lambda_{22}^0 &= 2 \left(\frac{1}{E_2} - \frac{\nu_{21}^2}{E_1} \right) \\ \Lambda_{44}^0 &= \frac{1}{G_{12}} \end{aligned} \quad (2)$$

The mode I and mode II components of the energy release rate can be obtained for the T-direction using Eq. (12) with $\eta_i = 1$

$$\begin{aligned} G_I(T) &= \frac{\pi a_0}{2} \Lambda_{22}^0 \sigma_{22}^2 \\ G_{II}(T) &= \frac{\pi a_0}{2} \Lambda_{44}^0 \tau_{12}^2 \end{aligned} \quad (3)$$

The corresponding components of the fracture toughness are given as:

$$\begin{aligned} G_{Ic}(T) &= \frac{\pi a_0}{2} \Lambda_{22}^0 (Y_{is}^T)^2 \\ G_{IIc}(T) &= \frac{\pi a_0}{2} \Lambda_{44}^0 (S_{is}^L)^2 \end{aligned} \quad (4)$$

where Y_{is}^T and S_{is}^L are the in-situ transverse tensile and shear strengths, respectively.

For propagation in the longitudinal direction, the mode I and mode II components of the energy release rate are:

$$\begin{aligned} G_I(L) &= \frac{\pi a_0}{4} \Lambda_{22}^0 \sigma_{22}^2 \\ G_{II}(L) &= \frac{\pi a_0}{4} \Lambda_{44}^0 \tau_{12}^2 \end{aligned} \quad (5)$$

and the components of the fracture toughness are:

$$\begin{aligned} G_{Ic}(L) &= \frac{\pi a_0}{4} \Lambda_{22}^0 (Y_{is}^T)^2 \\ G_{IIc}(L) &= \frac{\pi a_0}{4} \Lambda_{44}^0 (S_{is}^L)^2 \end{aligned} \quad (6)$$

Failure Criterion for Matrix Cracking

Having obtained expressions for the components of the energy release rate and fracture toughness, a failure criterion can be applied to predict the propagation of the slit crack represented in Fig. 3. Under the presence of both in-plane shear and transverse tension, the critical energy release rate G_c depends on the combined effect of all microscopic energy absorbing mechanisms such as the creation of new fracture surface. Relying on microscopic examinations of the fracture surface, Hahn [18] observed that the fracture surface topography strongly depends on the type of loading. With increasing proportion of the stress intensity factor K_{II} , more hackles are observed in the matrix, thereby indicating more energy absorption associated with crack extension. Therefore, Hahn proposed a mixed mode criterion written as a first-order polynomial in the stress intensity factors K_I and K_{II} . Written in terms of the mode I and mode II energy release rates, the Hahn criterion is

$$(1-g) \sqrt{\frac{G_I(i)}{G_{Ic}(i)}} + g \frac{G_I(i)}{G_{Ic}(i)} + \frac{G_{II}(i)}{G_{IIc}(i)} = 1, \quad (7)$$

$i = T, L$

where the material constant g is defined identically from either Eq. 4 or 6 as:

$$g = \frac{G_{Ic}}{G_{IIc}} = \frac{\Lambda_{22}^0}{\Lambda_{44}^0} \left(\frac{Y_{is}^T}{S_{is}^L} \right)^2 \quad (8)$$

A failure index for matrix tension can be expressed in terms of the ply stresses and in-situ strengths Y_{is}^T and S_{is}^L by substituting either Eqs. 3-4 or 5-6 into the criterion in Eq. 7 to yield:

$$(1-g) \frac{\sigma_{22}}{Y_{is}^T} + g \left(\frac{\sigma_{22}}{Y_{is}^T} \right)^2 + \left(\frac{\tau_{12}}{S_{is}^L} \right)^2 = 1 \quad (9)$$

The criterion presented in Eq. 9, with both linear and quadratic terms of the transverse normal stress and a quadratic term of the in-plane shear stress, is similar to the criteria proposed by Hahn [18], Liu [19] (for transverse tension and in-plane shear), and Puck [20]. In addition, if $g=1$, Eq. 7 reverts to the linear version of the criterion proposed by Wu and Reuter [21] for the propagation of delamination in laminated composites:

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \quad (10)$$

Furthermore, using $g=1$, Eq. 9 reverts to the well-known Hashin criterion [1] for transverse matrix cracking under both in-plane shear and transverse tension, where the ply strengths are replaced by the in-situ strengths:

$$FI_M = \left(\frac{\sigma_{22}}{Y_{is}^T} \right)^2 + \left(\frac{\tau_{12}}{S_{is}^L} \right)^2 \leq 1 \quad (11)$$

Calculation of In-Situ Strengths

The failure criterion proposed in Eq. 9 can only be as accurate as the values of the in-situ strengths Y_{is}^T and S_{is}^L that are used. These strengths represent the averaged ply stresses at which transverse cracks grow unstably. As was described earlier, these strength values depend on ply configuration parameters such as ply thickness and ply toughness.

It is possible to perform experimental tests to measure the in-situ strengths of laminates.

However, such tests demand complex set-ups, especially for the determination of the in-situ shear strength, and are expensive to perform. It is therefore desirable to calculate the in-situ strengths from material properties measurable from standard test specimens. In the following sections, various ply configurations are examined and fracture mechanics idealizations are applied to relate the stresses at failure to the ply toughnesses.

Failure of Thick Embedded Plies

A thick ply is defined as one in which the length of the slit crack is much smaller than the ply thickness, $2a_0 \ll t$, as illustrated in Fig. 4. The minimum thickness for a thick ply depends on the material used. For E-glass/epoxy and carbon/epoxy laminates, Dvorak and Laws [15] calculated the transition thickness between a thin and a thick ply to be approximately 0.7 mm, or about 5 to 6 plies.

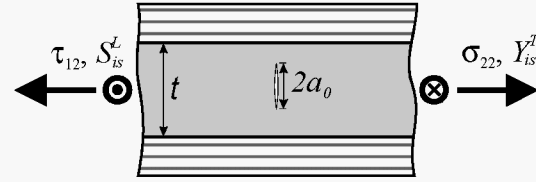


Fig. 4 – Geometry of slit crack in a thick embedded ply subjected to tension and shear loads.

For the geometry represented in Fig. 4, the crack can grow in the transverse or in the longitudinal direction. Comparing Eqs. 3 and 5, however, indicates that the energy release rate for the crack slit is twice as large in the transverse direction as it is in the longitudinal direction. Since Eq. 3 also indicates that the energy release rate is proportional to the crack length $2a_0$, the crack will grow unstably in the transverse direction. Once the crack reaches the constraining plies, it can propagate in the longitudinal direction, as well as induce a delamination.

The condition for crack propagation under biaxial loads is calculated using Eq. 9, where the in-situ strengths are obtained from the expressions of the fracture toughness in Eqs. 4:

$$\begin{aligned} Y_{is}^T &= \sqrt{\frac{2G_{Ic}(T)}{\pi a_0 \Lambda_{22}^0}} \\ S_{is}^L &= \sqrt{\frac{2G_{IIc}(T)}{\pi a_0 \Lambda_{44}^0}} \end{aligned} \quad (12)$$

It can be observed from Eqs. 12 that the in-situ strengths of thick plies Y_{is}^T and S_{is}^L are functions of the toughnesses $G_{Ic}(T)$ and $G_{IIc}(T)$ of the material and the size of the material flaw, $2a_0$. Therefore, the in-situ strengths for thick plies are independent of the ply thickness, as has been observed by Dvorak [15] and Leguillon [22], and as was shown in Fig. 1.

Failure of Thin Embedded Plies

Thin plies are defined as having a thickness smaller than the typical defect, $t < 2a_0$, so the slit crack extends across the entire thickness t of the ply, as represented in Fig. 5.

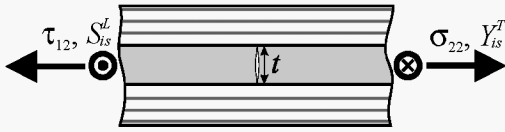


Fig. 5 – Geometry of slit crack in a thin embedded ply.

In the case of thin plies, crack defects can only grow in the longitudinal (L) direction, or trigger a delamination between the plies. The in-situ strengths for thin plies can be calculated from the components of the fracture toughness as:

$$\begin{aligned} Y_{is}^T &= \sqrt{\frac{8G_{Ic}(L)}{\pi t \Lambda_{22}^0}} \\ S_{is}^L &= \sqrt{\frac{8G_{IIc}(L)}{\pi t \Lambda_{44}^0}} \end{aligned} \quad (13)$$

where it can be observed that the in-situ strengths are inversely proportional to \sqrt{t} . The toughnesses $G_{Ic}(L)$ and $G_{IIc}(L)$ can be assumed to be the values measured by standard Fracture Mechanics tests, such as the double cantilever beam test (DCB) for mode I and the end notch flexure test (ENF) for mode II. Using Eqs. 12 and 13, Dvorak and Laws [15] obtained a good correlation between the predicted and

experimentally obtained in-situ strengths of both thick and thin 90° plies in $[0/90_n/0]$ laminates, as was shown in Fig. 1.

Failure of Unidirectional Laminates

The fracture of a unidirectional specimen is taken as a particular case of a thick ply [15]. The defect size is $2a_0$, as for thick embedded plies. However, in the absence of constraining plies, the critical initial slit crack is located at the surface of the laminate. For tensile loading, the crack can be located at the edge of the laminate, which increases the energy release rate when compared with a central crack. In the case of shear loading, there is no free edge, so the crack is a central crack, as shown in Fig. 6.

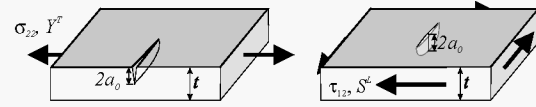


Fig. 6 – Unidirectional specimens under transverse tension and shear.

Crack propagation for unidirectional specimens subjected to tension or shear loads is obtained from the classic solution of the free edge crack [23].

$$\begin{aligned} G_{Ic}(T) &= 1.12^2 \pi a_0 \Lambda_{22}^0 (Y^T)^2 \\ G_{IIc}(T) &= \pi a_0 \Lambda_{44}^0 (S^L)^2 \end{aligned} \quad (14)$$

where Y^T and S^L are the material tensile and shear strengths as measured from unidirectional laminate tests. Note that the in-situ strengths for thick plies can be obtained as a function of the strength of unidirectional laminates by substituting Eqs. 14 into 12:

$$\begin{aligned} Y_{is}^T &= 1.12 \sqrt{2} Y^T \\ S_{is}^L &= \sqrt{2} S^L \end{aligned} \quad (15)$$

The failure criterion for unidirectional plies under in-plane shear and transverse tension is represented in Eq. 9. The toughness ratio g for a thick laminate can also be calculated in terms of the unidirectional properties by substituting Eqs. 14 into Eq. 8 to yield:

$$g = \frac{G_{Ic}}{G_{IIc}} = 1.12^2 \frac{\Lambda_{22}^0}{\Lambda_{44}^0} \left(\frac{Y^T}{S^L} \right)^2 \quad (16)$$

Failure of Outer Plies

Outer plies are taken as a special case of thin plies, as represented in Fig. 7. Following the procedure presented for a unidirectional laminate, it is considered that the stress intensity factor of an outer ply is larger than the stress intensity factor of a thin embedded ply. The relation between the stress intensity factors given by classical Fracture Mechanics solutions for free-edge cracks.

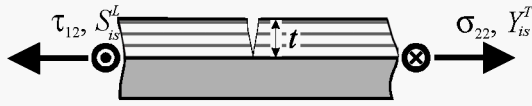


Fig. 7 – Geometry of slit crack in a thin outer ply.

In the case of thin outer plies, crack defects can only grow in the longitudinal (L) direction, or trigger a delamination between the plies. The in-situ strengths of thin outer plies can be calculated from the components of the fracture toughness as:

$$\begin{aligned} Y_{is}^T &= 1.79 \sqrt{\frac{G_{Ic}(L)}{\pi t \Lambda_{22}^0}} \\ S_{is}^L &= 2 \sqrt{\frac{G_{IIc}(L)}{\pi t \Lambda_{44}^0}} \end{aligned} \quad (17)$$

In-Situ Strength Factors

The in-situ matrix tensile and shear strength properties Y_{is}^T and S_{is}^L for composite plies are often expressed in terms of the measured unidirectional strengths Y^T and S^L using in-situ strength factors ω_I and ω_{II} that reflect the ply configuration. In general, the in-situ strengths are

$$\begin{aligned} Y_{is}^T &= \omega_I Y^T \\ S_{is}^L &= \omega_{II} S^L \end{aligned} \quad (18)$$

The unidirectional transverse tensile and in-plane shear strengths are determined from standard experimental tests. The fracture toughness for

crack growth in the longitudinal direction, for both mode I and mode II loading, is also required for the determination of the in-situ strengths.

The in-situ strength factors for thick embedded plies are given in Eqs. 15. The in-situ strength factors for thin embedded plies and for thin outer plies are calculated by substituting Eqs. 13 and 17 into Eq. 18. The in-situ factors for the different configurations are shown in Table 1.

Table 1 – In-situ Factors in Modes I and II

	Mode I	Mode II
Thick embedded ply	$\omega_I = 1.12\sqrt{2}$	$\omega_{II} = \sqrt{2}$
Thin embedded ply	$\omega_I = \frac{\sqrt{8}}{Y^T} \sqrt{\frac{G_{Ic}(L)}{\pi \Lambda_{22}^0 t}}$	$\omega_{II} = \frac{\sqrt{8}}{S^L} \sqrt{\frac{G_{IIc}(L)}{\pi \Lambda_{44}^0 t}}$
Outer ply	$\omega_I = \frac{1.78}{Y^T} \sqrt{\frac{G_{Ic}(L)}{\pi \Lambda_{22}^0 t}}$	$\omega_{II} = \frac{2}{S^L} \sqrt{\frac{G_{IIc}(L)}{\pi \Lambda_{44}^0 t}}$

The in-situ factors for the transverse tensile strength of T300/934 CFRP are presented in Fig. 8. The material properties were obtained by Dvorak and Laws [13].

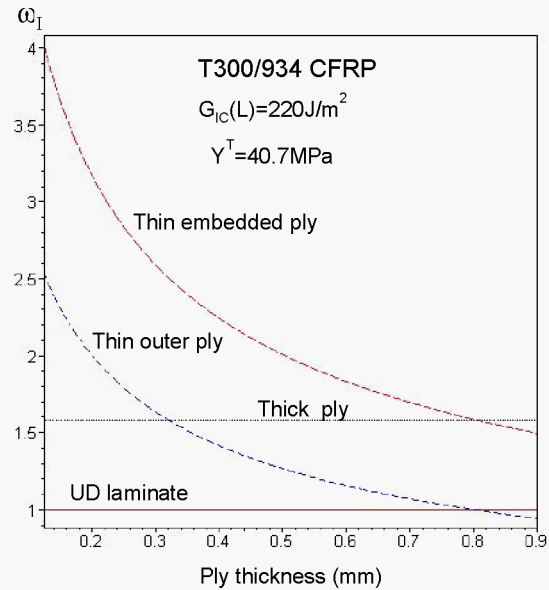


Fig. 8 – Predicted Mode I in-situ factors for T300/934 CFRP.

3. COMPARISON WITH EXPERIMENTS

A comparison of the strength of an AS4-55A carbon fiber reinforced ply predicted using Eq. 9 and published experimental results by Swanson [24] is shown in Fig. 9. The calculations were based on the material properties shown in Table 2. It can be observed that the Hashin criterion overpredicts the strength when shear is the dominant load. The results predicted using Eq. 9 are in overall good agreement with the experimental results.

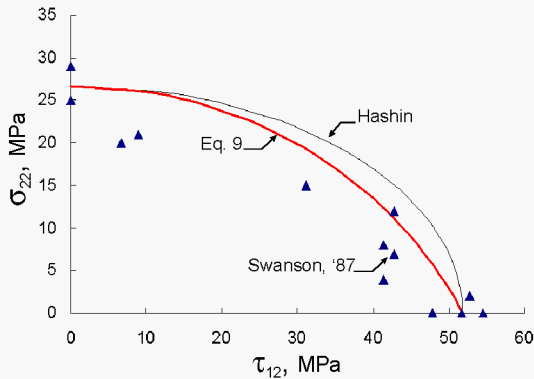


Fig. 9 – Comparison between calculated and experimental data for AS4/55A carbon/epoxy. (Swanson, Ref. [24].)

Table 2 – Properties of AS-4 55A in MPa. [24]

E_{11}	E_{22}	G_{12}	ν_{12}	Y^T	S^L
126,000	11,000	6,600	0.28	26.7	51.8

The strength of a glass/epoxy Scotch-Ply Type 1002 ply predicted using Eq. 9 is compared in Fig. 10 with published experimental results by Voloshin [25]. The calculations were based on the material properties shown in Table 3. It can be observed that the Hashin criterion overpredicts the strength when shear is the dominant load. The results predicted using Eq. 9 are in good agreement with the experimental results.

Table 3 – Properties of Scotch-ply E-glass in MPa. [25]

E_{11}	E_{22}	G_{12}	ν_{12}	Y^T	S^L
53,500	17,700	5,830	0.28	20.6	37.5

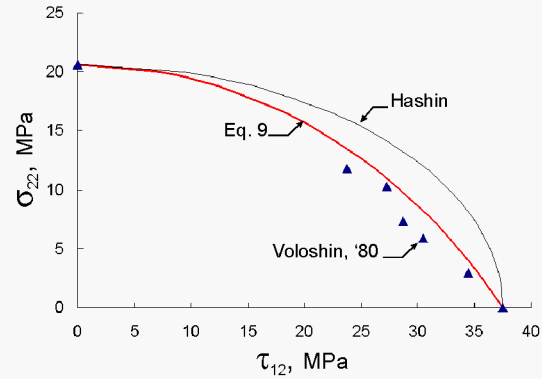


Fig. 10 – Comparison between calculated and experimental data for glass/epoxy Scotch-Ply Type 1002. (Voloshin [25])

4. DISCUSSION

Accurate failure criteria are necessary to assess the structural integrity of laminated composite structures. Transverse matrix cracking is a failure mode that is often considered benign and not given detailed consideration. However, transverse matrix cracks are the primary leakage path for gases in pressurized tanks. In the present paper, matrix cracking under combined transverse tension and inplane shear is discussed. In addition, the issue of the in-situ strength is examined by developing fracture mechanics idealizations of different ply configurations. The proposed fracture mechanics models relate the effective ply strength to the ply toughnesses, while accounting for the constraining effects of adjacent plies.

Failure envelopes were constructed for a graphite/epoxy material and a glass/epoxy material, and the predicted results indicate good correlation with experimental results.

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